Activity 4 Solutions:

By the end of this activity, you should have a better understanding of linear models. You should be able to identify the slope and y-intercept of each line.

Directions: Use the following equations and example to answer the questions.

Background: The target training heart rate is a useful measure of exercise intensity, which is used by everyone from athletes to patients recovering from heart attacks. This formula takes into consideration both the age of the person training as well as their resting heart rate.

Using this formula, the maximum heart rate ($HR_{\max}$) is computed as

$$HR_{\max} = 220 - A$$

where A represents the age of the person training. The reserve heart rate (HR_{rsv}) is computed as

$$HR_{\text{rsv}} = HR_{\max} - HR_{\text{rest}}$$

where HR_{rest} is the resting heart rate in beats per minute (bpm). Using these measures, the target heart rate (HR_{targ}) is computed as

$$HR_{\text{targ}} = p \cdot HR_{\text{rsv}} + HR_{\text{rest}}$$

where p is the training percentage. Depending on the specific training goals, p is commonly between 50% and 85% $(0.50 - 0.85)$ when exercising. In practice, lower values of p correspond to an intensity of exercise consistent with health maintenance and weight loss while higher values of p correspond to an exercise intensity compatible with intense fitness training.

Example: What is the approximate target heart rate for a 42 year old female whose resting heart rate is 80 bpm and who wishes to train at 65%?

Solution:

Here $A = 42$, so $HR_{max} = 220 - 42 = 178$.

Also $HR_{rest} = 80$bpm. Therefore $HR_{rsv} = HR_{max} - HR_{rest} = 178 - 80 = 98$bmp.

Since $p = 65\%$, the target heart rate for this individual is $HR_{targ} = 0.65HR_{rsv} + HR_{rest} = 0.65(98) + 80 = 63.7 + 80 = 143.7$ or approximately 144bpm.

1. What is the approximate target heart rate for a 66 year old male whose resting heart rate is 70 bpm and who wishes to train at 50%?

   Solution:

   Here $A = 66$, so $HR_{max} = 220 - 66 = 154$.

   Also $HR_{rest} = 70$bpm. Therefore $HR_{rsv} = HR_{max} - HR_{rest} = 154 - 70 = 84$bmp.

   Since $p = 50\%$, the target heart rate for this individual is $HR_{targ} = 0.5HR_{rsv} + HR_{rest} = 0.5(84) + 70 = 42 + 80 = 112$bpm.

2. The average male has a resting heart rate of about 70 bpm while the average female has a resting heart rate of about 75 bpm. Using this information, what is the approximate target heart rate for a 25 year old male and female training at 70%?

   Solution:

   Here $A = 25$, so $HR_{max} = 220 - 25 = 195$.

   For a male: $HR_{rest} = 70$bpm so $HR_{rsv} = HR_{max} - HR_{rest} = 195 - 70 = 125$bmp.

   Since $p = 70\%$, the target heart rate for this individual is $HR_{targ} = 0.7HR_{rsv} + HR_{rest} = 0.7(125) + 70 = 87.5 + 70 = 157.5$bpm.

   For a female: $HR_{rest} = 75$bpm so $HR_{rsv} = HR_{max} - HR_{rest} = 195 - 75 = 120$bmp.

   Since $p = 70\%$, the target heart rate for this individual is $HR_{targ} = 0.7HR_{rsv} + HR_{rest} = 0.7(120) + 75 = 84 + 75 = 159$bpm.

3. What is the resting heart rate of a 60 year old male whose heart rate is 124 bpm when training at 55%?

   Solution:

   Here $A = 60$ so $HR_{max} = 220 - 60 = 160$ and we know $HR_{targ} = 124$bpm.

   Since $p = 55\%$, the target heart rate for this individual is $HR_{targ} = 0.55HR_{rsv} + HR_{rest}$.

   Substituting $HR_{rsv} = HR_{max} - HR_{rest}$ into this equation we find

   $$\begin{aligned} 124 = HR_{targ} &= 0.55HR_{rsv} + HR_{rest} \\ &= .55(HR_{max} - HR_{rest}) + HR_{rest} \\ &= 0.55(160) - 0.55HR_{rest} + HR_{rest} \end{aligned}$$

   Therefore $124 - 88 = 0.55HR_{rest}$. We find $HR_{rest} = \frac{36}{0.45} = 80$bpm.

4. What is the resting heart rate of a 50 year old female whose heart rate is 132 bpm when training at 60%?

   Solution:

   Here $A = 50$ so $HR_{max} = 220 - 50 = 170$ and we know $HR_{targ} = 124$bpm.

   Since $p = 60\%$, the target heart rate for this individual is $HR_{targ} = 0.6HR_{rsv} + HR_{rest}$.

   Substituting $HR_{rsv} = HR_{max} - HR_{rest}$ into this equation we find

   $$\begin{aligned} 132 = HR_{targ} &= 0.6HR_{rsv} + HR_{rest} \\ &= .6(HR_{max} - HR_{rest}) + HR_{rest} \\ &= 0.6(170) - 0.6HR_{rest} + HR_{rest} \end{aligned}$$

   Therefore $132 - 102 = 0.6HR_{rest}$. We find $HR_{rest} = \frac{30}{0.4} = 75$bpm.

5. Lance Armstrong has a resting heart rate of 32 bpm. When he won the 2005 Tour de France, he was 33 years old.

   a) What was Armstrong's reserve heart rate when he won the 2005 Tour de France? **Solution:** Here $A = 33$ so $HR_{max} = 220 - 33 = 187$bpm. We also know $HR_{rest} = 32$bpm. Thus, $HR_{rsv} - HR_{rest} = 187 - 32 = 155$bpm.

   b) What was his target heart rate when he trained at 90%? **Solution:** Using the information above, we know $HR_{targ} = 0.9(155) + 32 = 139.5 + 32 = 171.5$bpm

6. Can you combine the 3 equations given at the beginning of the activity into 1 equation? Write the equation for target heart rate (HR_{targ}) in terms of age (A) and resting heart rate (HR_{rest}). **Solution:**

$$\begin{aligned} HR_{targ} &= p \cdot HR_{rsv} + HR_{rest} \\ &= p(HR_{max} - HR_{rest}) + HR_{rest} \\ &= p(220 - A) + (1 - p)HR_{rest} \end{aligned}$$

7. Using the equation you found in problem # 6, let the resting heart rate (HR_{rest}) be 60 bpm and let $p = 0.5$. Now, you have a linear equation where age is the independent variable and target heart rate is the dependent variable.

Solution:

Using what we found in problem 6, we know $HR_{targ} = 0.5(220 - A) + (1 - 0.5)(60) = 110 - 0.5A + 30 = -\frac{1}{2}A + 140$.

(a) What is the slope of the line?

Solution:

The slope of the line is $-\frac{1}{2}$

(b) What is the y-intercept of this line?

Solution:

The y-intercept of the line is $y = 140$.

(c) Sketch a graph of the line. The x-axis should be age and the y-axis is target heart rate.

8. Using the equation you found in problem # 6, let the resting heart rate (HR_{rest}) be 60 bpm and let $p = 0.75$. Now, you have a linear equation where age is the independent variable and target heart rate is the dependent variable.

Solution:

Using what we found in problem 6, we know $HR_{targ} = 0.75(220 - A) + (1 - 0.75)(60) = 165 - 0.75A + 15 = -\frac{3}{4}A + 180$.

(a) What is the slope of the line?

Solution:

The slope of the line is $-\frac{3}{4}$.

(b) What is the y-intercept of this line?

Solution:

The y-intercept of the line is $y = 180$.

(c) Sketch a graph of the line. The x-axis should be age and the y-axis is target heart rate.